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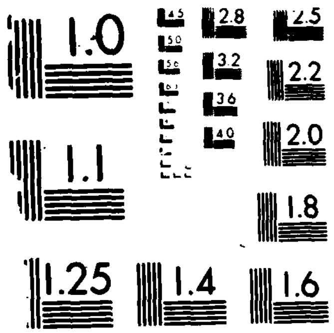

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Research Report CCS 551

A SEMI-INFINITE MULTICRITERIA PROGRAMMING APPROACH TO DATA ENVELOPMENT ANALYSIS WITH INFINITELY MANY DECISION-MAKING UNITS

by

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January 1987

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This research was partly supported by ONR Contract N00014-82-K-0295, and National Science Foundation Grants SES-8408134 and SES-8520806 with the Center for Cybernetic Studies, The University of Texas at Austin. Reproduction in whole or in part is permitted for any purpose of the United States Government.

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ABSTRACT

Via semi-infinite programming, Data Envelopment Analysis is extended to models with a (possibly) infinite number of Decision-Making Units in order to provide a simpler analytic structural base for forthcoming studies of statistical aspects. Relations with usual multicriteria problems are brought forth. In this paper, the only model considered is extension of phase one of the Charnes, Cooper, Thrall development of the CCR ratio model.

Key Words

Data Envelopment Analysis
Multicriteria Programming
Semi-infinite Programming
Pareto-Koopmans Optimality

A SEMI-INFINITE MULTICRITERIA PROGRAMMING APPROACH TO DATA ENVELOPMENT ANALYSIS WITH INFINITELY MANY DECISION-MAKING UNITS

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1. Introduction

DEA (Data Envelopment Analysis) models such as the CCR ratio model [4], the invariant multiplicative model [18] and the "additive" model [6] deal with the measurement of relative "technical" or "DEA" efficiency for a finite number of observed decision-making units (DMU's) and on the dual mathematical programming side with the estimation of a Pareto-optimal (= multi-criteria optimal) empirical economic frontier production function [3]. On the dual side, which is as important for the quantitative assessment of observed managerial peformance as for the in principle statistical estimation of the economic Pareto-optimal frontier function, one deals with an infinite set of in principle "possible" DMUs related to the observed ones by the defined structure of the model.

Economists in the past have dealt with frontier functions given in parametric form and have employed classical statistical techniques for estimating the parameters from sample observations assumed to be lying within infinite production possibility sets [20], [21], [22]. That DEA technique could also be effectively employed for these parametric determinations was pointed out for example in [3], but it was not until [6] that the DEA constraints and optimization problems were proved to be equivalent to the Charnes-Cooper test for Pareto-optimality of the observed DMU's which would equally well apply to any of the non-observed "possible DMU's."

The stage thus has now been set for fundamental extension of DEA efficiency analysis and, thereby Pareto-optimality economic function analysis, to an a priori infinite set of DMU's

specified in vastly increased variety than is provided by the accidents of structure of current models or the simple parametric forms currently employed for production functions. We here draw on the resources of semi-infinite programming [7, 8, ..., 14] to provide an elegant studied structure for this as well as a simple sound analytical base for forthcoming studies of stochastic aspects.

In this paper we limit ourselves to extensions via the CCR ratio model. In order not to incur non-Archimedean complications, we do this via the new multiphase (Archimedean) development of Charnes, Cooper & Thrall [19]. Thereby we achieve tractable, theoretical, and computational dual semi-infinite programming characterizations into which finite DMU sets can be embedded for study.

2. Model and Definition

We now formally introduce DEA via the following model. The model has an infinite number of DMUs (Decision Making Units) and m items of input and s outputs. Extending the CCR model [3] to a (possibly) infinite set of DMUs labeled by $x \in X$, we have:

CCR model to a (possibly) infinite set of DMUs labeled by $x \in X$, we have:

$$\text{Sup} \sum_{r=1}^{s} u_r g_r(x^0) \Big/ \sum_{i=1}^{m} v_i f_i(x^0)$$

$$\text{s.t.} \quad \sum_{r=1}^{s} u_r g_r(x) \Big/ \sum_{i=1}^{m} v_i f_i(x) \leq 1 \text{ for all } x \in X,$$

$$u = (u_1, \ldots, u_s)^T \geq 0,$$

$$v = (v_1, \ldots, v_m)^T \geq 0,$$

where $x^0 \in X$ is one of the DMU's,

$f_i(x)$ = the amount of the ith input of DMU x, $i = 1,2,\ldots,m$, $x \in X$,

$g_r(x)$ = the amount of the rth output of DMU x, $r = 1,2,\ldots,s$, $x \in X$,

X = the DMU set in, say, k-dimensional real space, R_k.

<u>Assumption (A1)</u>: For all $x \in R_k$

$$f(x) = (f_1(x), \ldots, f_m(x))^T > 0$$

and

$$g(x) = (g_1(x), \ldots, g_s(x))^T > 0$$

are continuous vector functions which are defined over R_k.

The above model is equivalent to the following model:

$$\begin{cases} \inf \sum_{i=1}^{m} v_i f_i(x^0) \Big/ \sum_{r=1}^{s} u_r g_r(x^0) \\ \text{s.t.} \quad \sum_{i=1}^{m} v_i f_i(x) \Big/ \sum_{r=1}^{s} u_r g_r(x) \geq 1 \text{ for all } x \in X, \\ \qquad u \geq 0, \\ \qquad v \geq 0. \end{cases}$$

Using vector notation, we can write

$$\begin{cases} \inf \; v^T f(x^0) / u^T g(x^0) \\ \text{s.t.} \quad v^T f(x) \; / \; u^T g(x) \geq 1 \text{ for all } x \in X, \\ \qquad u \geq 0, \\ \qquad v \geq 0. \end{cases}$$

Now we replace this nonconvex nonlinear fomulation with a semi-infinite programming problem by employing the Charnes-Cooper transformation of fractional programming [2].

$$t = 1 / u^T g(x^0), \quad w = tv, \quad \mu = tu,$$

to obtain the semi-infinite programming problem:

$$(P) \begin{cases} V_P = \inf w^T f(x^0) & \\ \text{s.t.} \; w^T f(x) + \mu^T[-g(x)] \geq 0 \text{ for all } x \in X, & (1a) \\ \qquad \mu^T g(x^0) = 1, & (1b) \\ \qquad \mu \geq 0, & (1c) \\ \qquad w \geq 0. & (1d) \end{cases}$$

The dual semi-infinite programming problem to (P) is

$$
(D)\quad \begin{cases}
V_D = \sup z_0. & \\
\text{s.t.} \quad \sum_{x \in \mathcal{X}} f(x)\,\lambda(x) \quad + s^- = f(x^0), & (2a) \\
\sum_{x \in \mathcal{X}} [-g(x)]\,\lambda(x) + s^+ = z_0\,[-g(x^0)], & (2b) \\
\lambda(x) \geq 0 \text{ for all } x \in \mathcal{X} & (2c) \\
s^-,\ s^+ \geq 0 & (2d)
\end{cases}
$$

where $\lambda(x) \in R_K$, $\lambda = [\lambda(x) : x \in \mathcal{X}] \in S$, the generalized finite sequence space (g.f.s.s.) [8], [9]. Namely, S is the vector space of all vectors $[\lambda(x): x \in \mathcal{X}]$ with only finitely many non-zero entries.

Theorem 1 [9], [14]. For the pair of Program (P) and (D) we have

$$V_P \geq V_D.$$

Assumption (A2): The set $\mathcal{X}$ is bounded and closed i.e., compact.

Lemma 1. Assuming (A1) and (A2), Programs (P) and (D) are consistent.

Proof: For Program (P), take

$$\mu^0 = g(x^0) / \| g(x^0) \|^2$$

and

$$w^0 = (w_1{}^0, 1, \ldots, 1)^T,$$

where

$$w_1{}^0 = \max_{x \in \mathcal{X}} \mu^{0T} g(x) / f_1(x) + 1.$$

Then

$$\mu^{0T} g(x^0) = 1,$$

$$w^{0T} f(x) + \mu^{0T}(-g(x^0)) > 0 \qquad \text{for all } x \in \mathcal{X},$$

$$w^0 > 0, \quad \mu^0 > 0.$$

Hence Program (P) is consistent.

For Program (D), take

$$\begin{aligned} \lambda^0(x) &= 0 \qquad \text{for all } x \in \chi, \\ s^{0-} &= f(x^0) > 0, \\ s^{0+} &= g(x^0) > 0, \\ z^0 &= -1, \end{aligned}$$

then $\lambda^0(x)$, s^{0-}, s^{0+}, z^0 is a feasible solution of Program (D). Hence Program (D) is consistent

Q.E.D.

Employing the "regularization" technique [1] we can obtain an Extended Dual Theorem.

Consider

$$(PM) \left\{ \begin{aligned} & V_{PM} = \inf \ (w^T f(x^0) + Mt) \\ & \text{s.t.} \qquad w^T f(x) + \mu^T [-g(x)] \geq 0 \text{ for all } X \in \chi \\ & \qquad\qquad t + \mu^T [-g(x^0)] \geq -1, \\ & \qquad\qquad t + \mu^T \ g(x^0) \geq 1, \\ & \qquad\qquad t \geq 0, \mu \geq 0, w \geq 0 \end{aligned} \right.$$

and

$$(DM) \left\{ \begin{aligned} & V_{DM} = \sup \ (Z_0'' - Z_0') \\ & \text{s.t.} \quad \sum_{x \in X} f(x)\ \lambda(x) + s^- = f(x^0), \\ & \qquad \sum_{x \in X} [-g(x)]\ \lambda(x) + s^+ = (Z_0'' - Z_0')[-g(x^0)], \\ & \qquad\qquad Z_0'' + Z_0' \leq M, \\ & \qquad\qquad \lambda(X) \geq 0 \quad \text{for all } X \in \chi, \\ & \qquad\qquad s^- \geq 0,\ s^+ \geq 0,\ Z_0'' \geq 0,\ Z_0' \geq 0, \end{aligned} \right.$$

where M is a large (unspecified) positive value.

The linear inequality system of (PM) is said to be "canonically closed" if the set of coefficients is compact in R_{m+s+2} and there exists interior points [9], [13].

Lemma 2. Assuming (A1) and (A2), the linear inequality system of (PM) is canonically closed.

Proof: According to the proof of Lemma 1, there exists (w^0, μ^0) such that

$$w^{0T} f(x) + \mu^{0T}[-g(x)] > 0 \quad \text{for all } x \in X,$$

$$\mu^{0T} g(x^0) = 1,$$

$$w^0 > 0, \quad \mu^0 > 0.$$

Let

$$t^0 = 1,$$

then (w^0, μ^0, t^0) is an interior point of the constraint set of program (PM).

It suffices to show that the set

$$W = \left\{ \begin{pmatrix} f(x) \\ -g(x) \end{pmatrix} : x \in X \right\}$$

is compact.

Let $\xi^k \in W$, $\lim_{k \to \infty} \xi^k = \xi^*$, then there are $x^k \in X$ such that

$$\xi^k = \begin{pmatrix} f(x^k) \\ -g(x^k) \end{pmatrix}.$$

According to Assumption (A2), there is a subsequence $\{x^{ki}\}$ of $\{x^k\}$ such that $\lim_{k \to \infty} x^{ki} = x^*$ and $x^* \in X$.

We can get (according to (A1))

$$\xi^* = \lim_{ki \to \infty} \xi^{ki} = \lim_{ki \to \infty} \begin{pmatrix} f(x^{ki}) \\ -g(x^{ki}) \end{pmatrix} = \begin{pmatrix} f(x^*) \\ -g(x^*) \end{pmatrix} \in X,$$

hence the set W is closed. Because of the continuity of $f(x)$ and $-g(x)$ and compactness of the set X, we know that W is bounded. Q E D

<u>Theorem 2</u> (Extended Dual Theorem) Assuming (A1) and (A2), the pair of Program (P) and (D) has the same value

$$V_P = \inf w^T f(x^0) = V_D = \sup Z_0.$$

Moreover, Program (D) assumes its supremum as a maximum.

Proof. By Lemma 1 and Lemma 2 and the Inhomogenous Haar Theorem [9],[13] we have

$$V_{PM} = V_{DM} = \max\ (Z_0'' - Z_0').$$

In Program (D) let

$$Z_0 = Z_0'' - Z_0', \quad Z_0'' \geq 0, \quad Z_0' \geq 0,$$

then we can get

$$(D)\left\{\begin{array}{l} V_D = \sup\ (Z_0'' - Z_0') \\ \text{s.t.} \quad \sum_{x \in X} f(x)\,\lambda(x) \quad + s- = f(x^0) \\ \quad \sum_{x \in X} [-g(x)]\,\lambda(x) \quad + s+ = (Z_0'' - Z_0')\,(-g(x^0)), \\ \quad \lambda(x) \geq 0 \quad \text{for all } x \in X, \\ \quad s^- \geq 0, \quad s^+ \geq 0, \quad Z_0'' \geq 0, \quad Z_0' \geq 0. \end{array}\right.$$

Comparing program (D) with program (DM), evidently

$$V_{DM} \leq V_D. \tag{2}$$

But by Theorem 1 we have

$$V_D \leq V_P. \tag{3}$$

Since Program (P) is consistent, we know that Program (PM) has a feasible solution with

$t = 0$, so

$$V_{PM} = V_P. \tag{4}$$

By (1), (2), (3) and (4), we have

$$V_P \geq V_D \geq V_{DM} = V_{PM} = V_P$$

Therefore

$$V_P = V_D.$$

Q.E.D.

Assumption (A3) There exists an optimal solution (w^0, μ^0) to Program (P).

From Assumptions (A1), (A2), (A3) and Theorem 2 we have that Programs (P) and (D) have respectively optimal solutions (w^0, μ^0) and $\lambda^0(x)$ for all $x \in X$, s^{0-}, s^{0+}, z_0^0. Following the arguments of Charnes, Gribik and Kortanek [15] we obtain similarly the complementary slackness results.

<u>Theorem 3</u>. Assuming (A1), (A2), and (A3), let (w^0, μ^0) be an optimal solution of Program (P) and $(\lambda^0(x)$ for all $x \in X$, s^{0-}, s^{0+}, $z_0^0)$ an optimal solution of Program (D).

Then

$$(w^{0T} f(x) + \mu^{0T}[-g(x)])\, \lambda^0(x) = 0 \quad \text{for all } x \in X, \tag{5}$$

and

$$\Big(\sum_{x \in X} f_i(x)\, \lambda^0(x) - f_i(x^0)\Big) w_i^0 = 0 \quad \text{for } i = 1, 2, \ldots, m, \tag{6}$$

$$\Big(\sum_{x \in X} [-g_r(x)]\, \lambda^0(x) - z_0^0[-g_r(x^0)]\Big) \mu_r^0 = 0 \quad \text{for } r = 1, 2, \ldots, s. \tag{7}$$

Proceeding further from (1a) and (1b) we have

$$v_P = \inf\ w^T f(x^0) \geq 1.$$

Now we give the following definitions.

<u>Definition 1</u>. Let $x^0 \in X$. DMU x^0 is said to be "DEA-efficient" if there is an optimal solution (w^0, μ^0) of Program (P) such that

$$v_P = w^{0T} f(x^0) = 1$$

and

$$w^0 > 0, \qquad \mu^0 > 0$$

<u>Definition 2</u>. Let $x^0 \in X$. DMU x^0 is said to be "weak DEA-efficient" if there is an optimal solution

(w^0, μ^0) of Program (P) such that

$$v_P = w^{0T} f(x^0) = 1$$

<u>Theorem 4</u>. Assuming (A1), (A2) and (A3), DMU x^0 is weak DEA-efficient if and only if Program (D) has an optimal solution $\lambda^0(x)$ for all $x \in X$, s^{0-}, s^{0+}, z_0^0 such that $z_0^0 = 1$

Proof. The result of the theorem is an immediate consequence of Theorem 2.

Q.E.D.

Assumption (A4): (Strict complementarity) If $s^{o-} = 0$, $s^{o+} = 0$ in all optimal solutions of Program (D), then there always exists an optimal solution (w^o, μ^o) of Program (P) such that

$$w^o > 0, \quad \mu^o > 0.$$

Theorem 5: Assuming (A1), (A2), (A3) and (A4), DMU x^o is DEA-efficient if and only if $V_D = 1$ and $s^{o-} = 0$, $s^{o+} = 0$ in all optimal solutions of Program (D).

Proof: Because of (A4), the result of the theorem is an immediate consequence of Theorem 2

Q.E.D.

We introduce a "non-Archimedean quantity" ϵ [4]. The symbol ϵ represents the infinitesimal; ϵ is less than every positive real number but greater than zero (in the extended real field).

Consider

$$\begin{cases} \min \; v^T f(x^o) \; / \; u^T g(x^o) \\ \text{s.t.} \\ v^T f(x) \; / \; u^T g(x) \geq 1 \quad \text{for all } x \in X. \\ (u^T g(x^o))^{-1} v^T \geq \epsilon \cdot \hat{e}^T > 0, \\ (u^T g(x^o))^{-1} u^T \geq \epsilon \cdot e^T > 0, \end{cases}$$

where

$$\hat{e}^T = (1, 1, \ldots, 1) \in R_m,$$

$$e^T = (1, 1, \ldots, 1) \in R_s.$$

Similarly, letting

$$t = 1 / u^T g(x_o), \qquad w = tv, \mu = tu,$$

we obtain the dual pair of Program (P') and Program (D'):

$$(\mathrm{P}')\quad \begin{cases} \min \quad w^T f(x^0) & \\ \text{s.t.} \quad w^T f(x) + \mu^T[-g(x)] \geq 0 \quad \text{for all } x \in X, & (1a)' \\ \qquad \mu^T g(x^0) = 1, & (1b)' \\ \qquad w^T \geq \epsilon \cdot \hat{e}^T & (1c)' \\ \qquad \mu^T \geq \epsilon \cdot e^T & (1d)' \end{cases}$$

and

$$(\mathrm{D}')\quad \begin{cases} \max\,(z_0 + \epsilon \cdot \hat{e}^T s^- + \epsilon \cdot e^T s^+) & \\ \text{s.t.} \quad \sum_{x \in X} f(x)\,\lambda(x) + s^- = f(x^0), & (2a)' \\ \qquad \sum_{x \in X} [-g(x)]\lambda(x) + s^+ = z_0(-g(x^0)), & (2b)' \\ \qquad \lambda(x) \geq 0 \quad \text{for all } x \in X, & (2c)' \\ \qquad s^-,\ s^+ \geq 0. & (2d)' \end{cases}$$

For the pair of Program (P') and (D'), the Extended Dual Theorem is also true. Suppose $(\lambda^0(x), s^0-, s^0+, z_0{}^0)$ is an optimal solution of Program (D') and that (A1), (A2), (A3) and (A4) hold. Paralleling Theorem 5 we define the following Definition 3 and Definition 4 which are respectively equivalent to Definition 1 and Definition 2.

Definition 3 Let $x^0 \in X$. DMU x^0 is said to be DEA efficient if an optimal solution $(\lambda^0(x), s^0-, s^0+, z_0{}^0)$ of Program (D') satisfies $z_0{}^0 = 1$ and $s^0- = 0$, $s^0+ = 0$.

Definition 4 Let $x^0 \in X$. DMU x^0 is said to be weak DEA efficient if an optimal solution $(\lambda^0(x), s^0-, s^0+, z_0{}^0)$ of Program (D') satisfies $z_0{}^0 = 1$.

3. DEA Efficiency and Pareto Efficiency

For efficient production we wish to maximize on outputs while minimizing on inputs, so we consider the following multiobjective mathematical programming problem:

$$(VP)\quad \begin{cases} v - \min\ (f_1(x), \ldots, f_m(x), -g_1(x), \ldots, -g_s(x)) \\ \text{s.t.}\quad x \in X \end{cases}$$

where $F(x) = (f_1(x), \ldots, f_m(x))^T$ is the input vector function and $g(x) = (g_1(x), \ldots, g_s(x))^T$ is the output vector function defined for X in R_K.

<u>Definition 5</u> Let $x^0 \in X$. x^0 is said to be "Pareto efficient" for the multiobjective programming problem (VP) if there is no $x \in X$, $x \neq x^0$ such that

$$(f(x), -g(x)) \leq (f(x^0), -g(x^0))$$

with at least one strict inequality.

<u>Definition 6</u> Let $x^0 \in X$. x^0 is said to be weak Pareto efficient for the multiobjective programming problem (VP) if there is no $x \in X$ such that

$$(f(x), -g(x)) < (f(x^0), -g(x^0)).$$

In this section we study the relation between (weak) DEA efficiency and (weak) Pareto efficiency.

<u>Lemma 3</u> If the optimal solution (w^0, μ^0) of Program (P) satisfies

$$V_P = w^{0T} f(x^0) = 1,$$

then x^0 is an optimal solution of the following problem

$$\min_{x \in X}\ (w^{0T} f(x) + \mu^{0T}[-g(x)]).$$

Proof: Since $w^{0T} f(x^0) = 1$

and

$$w^{0T} f(x) + \mu^{0T}[-g(x)] \geq 0 \quad \text{for all } x \in X,$$

$$\mu^{0T} g(x^0) = 1.$$

we have that for all $x \in X$

$$w^{oT}f(x) + \mu^{oT}[-g(x)] \geq 0 = w^{oT}f(x^o) + \mu^{oT}(-g(x^o)).$$

Q.E.D.

Lemma 4. Assuming (A1), (A2) and (A3), and if the optimal solution (w^o,μ^o) of the Program (P) satisfies

$$V_p = w^{oT}f(x^o) > 1,$$

then there exists some $x^* \in X$ such that

$$w^{oT}f(x^*) + \mu^{oT}[-g(x^*)] = 0.$$

Proof: Suppose that for all $x \in X$

$$w^{oT}f(x) + \mu^{oT}[-g(x)] > 0.$$

Without loss of generality, suppose $w_1^o > 0$. We take

$$\Delta w_1 = \min\{ w_1^o, \min_{x \in X} w^{oT}f(x) + \mu^{oT}[-g(x)] / f_1(x)\} > 0,$$

thus $w_1^o - \Delta w_1 \geq 0$ and for all $x \in X$

$$(w_1^o - \Delta w_1) f_1(x) + \sum_{i=2}^{m} w_i^o f_i(x) + \mu^{oT} [-g(x)]$$

$$= w^{oT}f(x) + \mu^{oT}[-g(x)] - \Delta w_1 f_1(x)$$

$$\geq 0.$$

Namely, $((w_1^o - \Delta w_1, w_2^o, \ldots, w_m^o)^T, \mu^o)$ is a feasible solution of Program (P). But we have

$$(w_1^o - \Delta w_1, w_2^o, \ldots, w_m^o) f(x^o)$$

$$= w^{oT}f(x^o) - \Delta w_1 f_1(x^o)$$

$$< w^{oT} f(x^o)$$

which yields a contradiction.

QED

Theorem 6. Assuming (A1), (A2) and (A3), and (w^0, μ^0) is an optimal solution of Program (P), then

$$V_P = w^{0T} f(x^0) = 1$$

if and only if x^0 is an optimal solution of the following problem

$$\min_{x \in X} (w^{0T} f(x) + \mu^{0T} [-g(x)].$$

Proof. According to Lemma 3, the necessity is true. Now we prove the sufficient part of the theorem. Suppose that x^0 is an optimal solution of the problem

$$\min_{x \in X} (w^{0T} f(x) + \mu^{0T} [-g(x)],$$

where (w^0, μ^0) is an optimal solution of (P), but

$$V_P = w^{0T} f(x^0) > 1$$

From Lemma 4 there exists $x^* \in X$ such that

$$w^{0T} f(x^*) + \mu^{0T}[-g(x^*)] = 0$$

Since

$$w^{0T} f(x^0) + \mu^{0T}[-g(x^0)]$$

$$= w^{0T} f(x^0) - 1$$

$$> 0,$$

we have that

$$w^{0T} f(x^0) + \mu^{0T} [-g(x^0)] > 0 = w^{0T} f(x^*) + \mu^{0T} [-g(x^*)]$$

which yields a contradiction.

Q.E.D.

Theorem 7 Assuming (A1), (A2), and (A3), if the optimal solution (w^0, μ^0) of Program (P) satisfies

$$V_P = w^{0T} f(x^0) = 1$$

then

(i) x^0 is weak Pareto efficient for the multiobjective programming

problem (VP).

(ii) If for all $x \in X$, $x \neq x^0$,

$$w^{0T} f(x) + \mu^{0T} [-g(x)] > 0,$$

then x^0 is Pareto efficient for the multiobjective programming problem (VP).

(iii) If $w^0 > 0$, $\mu^0 > 0$, then x^0 is Pareto efficient for the multiobjective

prograpprogramming problem (VP).

Proof: If there exists $\bar{x} \in X$, $\bar{x} \neq x^0$ such that

$$(f(\bar{x}), \; -g(\bar{x})) < (f(x^0), \; -g(x^0)) \; ,$$

then from $w^0 \geq 0$, $\mu^0 \geq 0$ we have

$$w^{0T} f(\bar{x}) + \mu^{0T} [-g(\bar{x})] < w^{0T} f(x^0) + \mu^{0T} [-g(x^0)].$$

On the other hand, according to Theorem 6, x^0 is an optimal solution of the following problem

$$\min_{x \in X} \; (w^{0T} f(x) + \mu^{0T} [-g(x)]),$$

which yields a contradiction. We have thereby completed the proof of (i).

If there exists $\bar{x} \in X$, $\bar{x} \neq x^0$ such that

$$(f(\bar{x}), \; -g(\bar{x})) \leq (f(x^0), \; -g(x^0)) \; ,$$

then from $w^0 \geq 0$, $\mu^0 \geq 0$ we have

$$w^{0T} f(\bar{x}) + \mu^{0T} [-g(\bar{x})] \leq w^{0T} f(x^0) + \mu^{0T} [-g(x^0)].$$

From Theorem 6 we must have

$$w^{0T} f(\bar{x}) + \mu^{0T} [-g(\bar{x})] = w^{0T} f(x^0) + \mu^{0T} [-g(x^0)]. \quad (8)$$

On the other hand, from the assumption of case (ii), we have

$$w^{0T} f(\bar{x}) + \mu^{0T} [-g(\bar{x})] > 0$$

and

$$w^{0T} f(x^0) + \mu^{0T} [-g(x^0)] = 0$$

which contradicts (8). We have therefore completed the proof of (ii).

If there exists $\bar{x} \in X$, $\bar{x} \neq x^0$ such that

$$(f(\bar{x}), \; -g(\bar{x})) \leq (f(x^0), \; -g(x^0)) \; ,$$

then from $w^0 > 0$, $\mu^0 > 0$ we have

$$w^{0^T} f(\bar{x}) + \mu^{0^T} [-g(\bar{x})] < w^{0^T} f(x^0) + \mu^{0^T} [-g(x^0)]$$

which yields a contradiction. We have thus proved (iii).

Q.E.D.

From Definition 1, ..., Definition. 4 and Theorem 7 we have Corollary 1 and Corollary 2.

Corollary 1. Assuming the (A1), (A2), (A3) and (A4), we have

(i) If $x^0 \in X$ is weak DEA efficient, then x^0 is also weak Pareto efficient for the multiobjective programming problem (VP).

(ii) If $x^0 \in X$ is DEA efficient, then x^0 is also Pareto efficient for the multiobjective programming problem (VP).

Corollary 2. If $(w^{0^T}, \mu^{0^T}) \geq 0$, $\mu^0 \neq 0$ satisfies

$$w^{0^T} f(x) + \mu^{0^T} [-g(x)] \geq 0 \text{ for all } x \in X$$

and $x^* \in X$ such that

$$w^{0^T} f(x^*) + \mu^{0^T} [-g(x^*)] = 0,$$

then

(i) x^* is weak Pareto efficient for the multiobjective programming problem (VP).

(ii) If for all $x \in X$, $x \neq x^*$ satisfies

$$w^{0^T} f(x) + \mu^{0^T} [-g(x)] > 0$$

then x^* is Pareto efficient for the multiobjective programming problem (VP).

(iii) If $w^0 > 0$, $\mu^0 > 0$, then x^* is Pareto efficient for the multiobjective programming problem (VP).

Proof: Let

$$w^* = w^0 / \mu^{0^T} g(x^*),$$

and

$$\mu^* = \mu^0 / \mu^{0^T} g(x^*),$$

then

$$w^{*T}f(x) + \mu^{*T}[-g(x)] \geq 0 \quad \text{for all } x \in X,$$
$$\mu^{*T}g(x^*) = 1,$$
$$w^* \geq 0, \quad \mu^* \geq 0,$$

and

$$w^{*T}f(x^*) + \mu^{*T}[-g(x^*)] = 0 .$$

Since $w^{*T}f(x^*) = \mu^{*T}g(x^*) = 1$ we have that (w^*, μ^*) is an optimal solution of the following program,

$$\min \ w^T f(x^*)$$
$$\text{s.t.} \quad w^T f(x) + \mu^T[-g(x)] \geq 0 \quad \text{for all } x \in X,$$
$$\mu^T g(x^*) = 1,$$
$$w \geq 0, \quad \mu \geq 0 .$$

According to Theorem 7, the results (i), (ii) and (iii) are true

Q.E.D.

Theorem 8 Assuming (A1), (A2), (A3) and (A4), if $(\lambda^0(x)$ for all $x \in X$, s^{0-}, s^{0+}, $z_0^0)$ is an optimal solution of Program (D'), then

(i) If $z_0^0 = 1$ then x^0 is weak Pareto efficient for the multiobjective programming problem (VP).

(ii) If $z_0^0 = 1$ and $s^{0-} = 0$, $s^{0+} = 0$, then x^0 is Pareto efficient for the multiobjective programming problem (VP)

Proof According to assumption (A4), Theorem 5 and Theorem 7 ((i) and (iii)), the results of the theorem are true

Q.E.D.

In general, Pareto efficiency for the multiobjective programming problem (VP) is not necessarily DEA efficiency even for the case of a finite number of DMU's, for example, we

consider a DEA model which has 2 DMU's and a single output and single input. Figure 1 provides an illustration in which 2 DMU's are represented by points z^1 and z^2, where

$$f(z) = x_1, \quad g(z) = y_1$$

and

$$\chi = \{z^1, z^2\} = \{(4, 3), (3, 1)\} .$$

Now to test the efficiency of z^1, consider the program:

$$\begin{cases} \min 4w \\ \text{s.t. } 4w - 3\mu \geq 0 \\ \quad 3w - 1\mu \geq 0 \\ \quad\quad 3\mu = 1 \\ \quad w \geq 0, \quad \mu \geq 0 \end{cases}$$

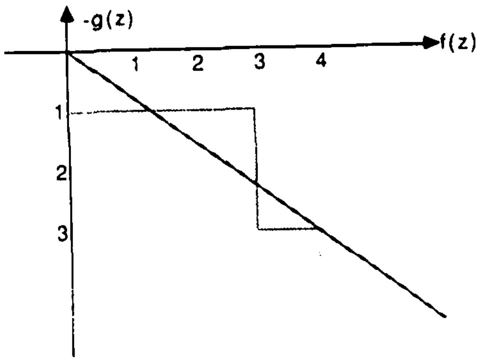

Figure 1

we find as optimal solution

$$w^o = 1/4 > 0, \quad \mu^o = 1/3 > 0$$

and

$$V_p = 1.$$

From Definition 1, DMUz^1 is DEA efficient.

Now to test the efficiency of z^2, consider the program.

$$\begin{cases} \min 3w \\ \text{s.t. } 4w - 3\mu \geq 0 \\ \quad 3w - 1\mu \geq 0 \\ \quad\quad 1\mu = 1 \\ \quad w \geq 0, \quad \mu \geq 0 \end{cases}$$

we find the optimal solution

$$w^o = 3/4 > 0, \quad \mu^o = 1 > 0$$

and

$$V_p = 9/4 > 1.$$

Therefore DMUz^2 is not DEA efficient.

Now the corresponding multiobjective programming problem is

$$(VP)\quad \begin{cases} v - \min\ (x_1, -y_1) \\ \\ \text{s.t. } z = (x^1, y^1) \in \{(4, 3), (3, 1)\} \end{cases}$$

From Figure 1 we know that z^1 and z^2 are Pareto efficient for the multiobjective programming problem (VP).

Let

$$C(\chi) = \{\sum_{j=1}^{k} \lambda_j z^j : z^j \in \chi, \lambda_j \geq 0, j = 1, 2, \ldots, K, \quad k \geq 1\}$$

We know that $C(\chi)$ is a convex cone. Now we consider the multiobjective programming problem:

$$(VP')\quad \begin{cases} V - \min\ (f_1(x), \ldots, f_m(x), -g_1(x), \ldots, -g_s(x)) \\ \\ \text{s.t. } x \in C(\chi) \end{cases}$$

and study the relation between DEA efficiency and Pareto efficiency of the multiobjective programming problem (VP′).

Lemma 5. Assuming (A1) and (A2), then the following linear inequality system

$$\begin{cases} w^T f(x) + \mu^T [-g(x)] \geq 0 \text{ for all } x \in \chi, \\ w^T \qquad\qquad\qquad \geq \hat{e}^T, \\ \qquad\qquad \mu^T \qquad \geq e^T \end{cases}$$

is "canonically closed."

Proof. By using a method similar to that in Lemma 2, we can prove that the set of coefficients of the linear inequality system is compact and there exists an interior point in the linear inequality system.

Q.E.D.

Assumption (A5): $f(x)$ and $g(x)$ are linear vector functions which are defined over R_K

Assumption (A6): There exists an optimal solution to the semi-infinite program:

$$\begin{cases} \min\ (w^T f(x^0) + \mu^T [-g(x^0)]) \\ \text{s.t.}\quad w^T f(x) + \mu^T [-g(x)] \geq 0 \text{ for all } x \in X \\ \qquad w^T \qquad\qquad\qquad\qquad \geq \hat{e}^T, \\ \qquad\qquad\qquad \mu^T \qquad\qquad \geq e^T. \end{cases}$$

Theorem 9. Assuming (A1), (A2), (A3), (A4), (A5) and (A6), if $x^0 \in X$ is Pareto efficient for the multiobjective programming problem (VP´), then DMU x^0 is DEA efficient.

Proof. Since $x^0 \in X$ is Pareto efficient for (VP´), the following linear inequality system

$$(I)\quad \begin{cases} \sum\limits_{x \in X} (f(x), -g(x))\lambda(x) \leq (f(x^0), -g(x^0)) \\ \lambda(x) \geq 0, \text{ for all } x \in X \\ \lambda = [\lambda(x) : x \in X] \in S \text{ (see [8])} \end{cases}$$

is inconsistent. In fact, if there exists a solution $\lambda^* = [\lambda^*(x) : x \in X] \in S$, then according to (A5) we have

$$f(\sum_{x \in X} \lambda^*(x) \cdot x) = \sum_{x \in X} \lambda^*(x)\ f(x)$$

and

$$-g(\sum_{x \in X} \lambda^*(x) \cdot x) = \sum_{x \in X} \lambda^*(x)\ [-g(x)],$$

thus

$$\begin{pmatrix} f(\sum\limits_{x \in X} \lambda^*(x) \cdot x) \\ \\ -g(\sum\limits_{x \in X} \lambda^*(x) \cdot x) \end{pmatrix} = \begin{pmatrix} \sum\limits_{x \in X} \lambda^*(x)\ f(x) \\ \\ \sum\limits_{x \in X} \lambda^*(x)\ [-g(x)] \end{pmatrix} \leq \begin{pmatrix} f(x^0) \\ \\ -g(x^0) \end{pmatrix}$$

Since $\sum_{x \in X} \lambda^*(x) \cdot x \in C(X)$, then x^0 is not Pareto efficient for (VP´).

We consider the pair of Program $(\bar{P})$ and Program $(\bar{D})$.

$$(\bar{P})\quad \begin{cases} \min\ (w^T f(x^0) + \mu^T [-g(x^0)]) \\ \text{s.t.}\quad w^T f(x) + \mu^T [-g(x)] \geq 0 \text{ for all } x \in X, \\ \qquad w^T \qquad\qquad\qquad \geq \hat{e}^T, \\ \qquad\qquad\qquad \mu^T \quad \geq e^T \end{cases}$$

and

$$(\bar{D})\quad \begin{cases} \max\ (\hat{e}^T s^- + e^T s^+) \\ \text{s.t.}\quad \sum_{x\in X} f(x)\,\lambda(x) + s^- = f(x^0), \\ \qquad \sum_{x\in X} [-g(x)]\lambda(x) + s^+ = [-g(x^0)] \\ \qquad \lambda(x) \geq 0 \quad \text{for all } x \in X \\ \qquad s^- \geq 0, \quad s^+ \geq 0. \end{cases}$$

where $\quad [\lambda(x) : x \in X] \in S.$

According to Lemma 5 and the Inhomogeneous Haar theorem (see Theorem 2), there are optimal solutions for Program $(\bar{P})$ and Program $(\bar{D})$ and

$$\min\ (w^T f(x^0) + \mu^T[-g(x^0)]) = \max\ (\hat{e}^T s^- + e^T s^+) = 0$$

(since $(_I)$ is inconsistent, the optimal value $\max\ (\hat{e}^T s^- + e^T s^+) = 0$).

Let (w^0,μ^0) be an optimal solution of Program $(\bar{P})$, then $w^0 > 0$, $\mu^0 > 0$ and for all $x \in X$

$$w^{0T} f(x) + \mu^{0T}[-g(x)] \geq 0 = w^{0T} f(x^0) + \mu^{0T}[-g(x^0)]. \tag{9}$$

Let

$$w^* = w^0 / \mu^{0T} g(x^0)\ , \qquad \mu^* = \mu^0 / \mu^{0T} g(x^0)\ , \tag{10}$$

from (9) and (10) we can get

$$w^{*T} f(x) + \mu^{*T}[-g(x)] \geq 0 \qquad \text{for all } x \in X,$$

$$\mu^{*T} g(x^0) = 1,$$

$$w^* > 0, \qquad \mu^* > 0$$

and

$$w^{*T} f(x^0) = \mu^{*T} g(x^0) = 1.$$

Therefore (w^*, μ^*) is an optimal solution of the Program

$$\begin{cases} \min \; w^T f(x^0) \\ \text{s.t.} \quad w^T f(x) + \mu^T[-g(x)] \geq 0 \qquad \text{for all } x \in X \\ \qquad \mu^T g(x^0) = 1, \\ w \geq 0, \qquad \mu \geq 0 \end{cases}$$

and

$$\mu^* > 0, \; w^* > 0, \; w^{*T} f(x^0) = 1$$

According to the Definition 1, DMUx^0 is DEA efficient.

Q.E.D.

<u>Theorem 10</u> Assuming (A1), (A2), (A3), (A4), (A5) and (A6), if $x^0 \in X$ and DMUx^0 is DEA efficient, then x^0 is Pareto efficient for the multiobjective programming problem (VP').

Proof. According to Corollary 1 (ii), if DMUx^0 is DEA efficient, then x^0 is Pareto efficient for the multiobjective programming problem (VP). Now we prove that x^0 is also Pareto efficient for the multiobjective programming problem (VP').

From Theorem 6, we have that for all $x \in X$

$$w^{0T} f(x) + \mu^{0T}[-g(x)] \geq w^{0T} f(x^0) + \mu^{0T}[-g(x^0)] = 0$$

Thus for all $x \in C(X)$ we can get

$$w^{0T} f(x) + \mu^{0T} [-g(x)]$$

$$= w^{0T} f(\sum_{j=1}^{k} \lambda_j x^j) + \mu^{0T} [-g(\sum_{j=1}^{k} \lambda_j x^j)]$$

$$= w^{0T} (\sum_{j=1}^{k} \lambda_j f(x^j)) + \mu^{0T} [-\sum_{j=1}^{k} \lambda_j g(x^j)]$$

$$= \sum_{j=1}^{k} (w^{0T} f(x^j) + \mu^{0T}[-g(x^j)])\lambda_j$$

$$\geq \sum_{j=1}^{k} (w^{0T} f(x^0) + \mu^{0T}[-g(x^0)])\lambda_j$$

$$= w^{0T} f(x^0) + \mu^{0T} (-g(x^0)),$$

where $x^j \in X$, $\lambda_j \geq 0$, $j = 1, 2, \ldots, k$.

Since $w^0 > 0$, $\mu^0 > 0$ we have that x^0 is Pareto efficient for the multiobjective programming problem (VP') (see the proof of Theorem 7 (iii)).

Q.E.D.

4 Case of a finite number of 'DMUs'

In Program (P) and Program (D) we take

$$X = \{Z : Z = (X_j, Y_j), j = 1, 2, \ldots, n\}$$

and

$$f(Z) = X, \qquad g(Z) = Y,$$

where $X_j \in R_m$, $Y_j \in R_s$, $j = 1, 2, \ldots, n$ and

$$Z = (X, Y) \quad \in X \subset R_{m+s}.$$

Thus we can get the pair of program (P) and (D) corresponding to the case of a finite number of DMUs:

$$(P) \begin{cases} \min \ w^T X_{j0} \\ \text{s.t.} \quad w^T X_j + \mu^T(-Y_j) \geq 0, \ j = 1, 2, \ldots, n \\ \qquad\qquad \mu^T Y_{j0} = 1. \\ \qquad w \geq 0, \mu \geq 0. \end{cases}$$

and

$$(D) \begin{cases} \max Z_0 \\ \text{s.t.} \quad \sum_{j=1}^{n} X_j \lambda_j + S^- = X_{j0}, \\ \qquad \sum_{j=1}^{n} (-Y_j)\lambda_j + S^+ = Z_0 (-Y_{j0}) \\ \qquad\qquad \lambda_j \geq 0, \ j = 1, \ldots, n, \\ \qquad\qquad S^- \geq 0, S^+ \geq 0, \end{cases}$$

where $j_0 \in \{1, 2, \ldots, n\}$.

Obviously, the pair of Program (P) and Program (D) are standard linear programming problems. So are $(\bar{P})$ and $(\bar{D})$. For standard linear programming, Assumptions (A1), (A2), (A3), (A4), (A5), (A6) naturally hold. So all of the conclusions we got in the former sections

certainly hold under the situation of a finite number of DMUs. Thus, we also can get the multiobjective programming problems (VP) and (VP′)

$$(VP)\quad \begin{cases} V-\min\,(x_1, x_2, \ldots, x_m, -y_1, -y_2, \ldots, -y_s) \\ \text{s.t.}\quad (X, Y) \in \{(X_j, Y_j) \quad j = 1, 2, \ldots, n\} \end{cases}$$

and

$$(VP')\quad \begin{cases} V-\min\,(x_1, x_2, \ldots, x_m, -y_1, -y_2, \ldots, -y_s) \\ \text{s.t.}\quad (X, Y) \in \{\sum_{j=1}^{n} (X_j, Y_j)\,\lambda_j \;:\; \lambda_j \geq 0,\ j = 1, \ldots, n\} \end{cases}$$

Theorem 11. For the case of a finite number of 'DMUs' we have

(i) if the j_0th DMU is (weak) DEA efficient, then (x_{j0}, y_{j0}) is (weak) Pareto efficient for the multiobjective programming problem (VP).

(ii) the j_0th DMU is DEA efficient if and only if (x_{j0}, y_{j0}) is Pareto efficient for the multiobjective programming problem (VP′).

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER CCS 551	2. GOVT ACCESSION NO. ADA180180	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) A SEMI-INFINITE MULTICRITERIA PROGRAMMING APPROACH TO DATA ENVELOPMENT ANALYSIS WITH INFINITELY MANY DECISION-MAKING UNITS		5. TYPE OF REPORT & PERIOD COVERED Technical 6. PERFORMING ORG. REPORT NUMBER CCS 551
7. AUTHOR(s) A. Charnes, W.W. Cooper, Q.L. Wei		8. CONTRACT OR GRANT NUMBER(s) N00014-82-K-0295
9. PERFORMING ORGANIZATION NAME AND ADDRESS Office of Naval Research (Code 434) Washington , D.C.		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS		12. REPORT DATE January 1987 13. NUMBER OF PAGES 28
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16 DISTRIBUTION STATEMENT (of this Report)

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17 DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18 SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Data Envelopment Analysis, Multicriteria programming, Semi-infinite programming; Pareto-Koopmans Optimality

20 ABSTRACT (Continue on reverse side if necessary and identify by block number)

Via semi-infinite programming, Data Envelopment Analysis is extended to models with a (possibly) infinite number of Decision-Making Units in order to provide a simpler analytic structural base for forthcoming studies of statistical aspects. Relations with usual multicriteria problems are brought forth. In this paper, the only model considered is extension of phase one of the Charnes, Cooper, Thrall development of the CCR ratio model.

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S/N 0102-014-6601

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